Econ 802

Final Exam

Greg Dow December 8, 2017

All questions have equal weight. If something is unclear, please ask.

1. Imagine that an undergraduate economics student asks you the following series of questions. Give clear verbal explanations (do not use math or graphs).

(a) "In the theory of the firm, economists often assume that the input requirement set is convex, and in the theory of the consumer they often assume the upper contour set is convex. Why are these assumptions useful?"

(b) "For production functions, economists rarely talk about increasing returns to scale in the context of price-taking behavior. Also, for utility functions, they rarely talk about increasing returns to scale at all. Why is that?"

(c) "Consumer theory uses the Slutsky equation, but the theory of the firm does not. Could there be a Slutsky equation for the firm? If so, why don't economists ever discuss it?"

2. The Acme Corporation has the production function $y = ax_1^{1/2} + bx_2$ where $y \geq 0$ is output and $x = (x_1, x_2) \geq 0$ are the inputs, with $a > 0$ and $b > 0$. The output price is $p > 0$ and the input prices are $w = (w_1, w_2) > 0$.

(a) When does Acme's profit maximization problem have a solution? When is the solution unique? When does the solution involve $x_1 = 0$ or $x_2 = 0$? Explain.

(b) In the short run, $x_2 > 0$ is fixed. Solve for the short run cost function $c(w, y, x_2)$. Describe Acme's short run supply function $y_{SR}(p)$ mathematically, show it on a graph, and give a verbal explanation.

(c) In the long run, both inputs are variable. Solve for the long run cost function $c(w, y)$. Describe Acme's long run supply function $y_{LR}(p)$ mathematically, show it on a graph, and give a verbal explanation.

3. The Log Lady has the utility function $u(x_1 \, . \, . \, x_n) = (1/n) \sum_i \ln x_i$. The price vector is $p = (p_1 \, . \, . \, p_n) > 0$ and income is $m > 0$.

(a) Solve for the vector of Marshallian demands $x(p, m)$. You can assume that first order conditions are sufficient and all solutions are interior.

(b) Solve for the indirect utility function $v(p, m)$ and the expenditure function $e(p, u)$.

(c) For a fixed $p > 0$, the associated income expansion path is the set IEP = $\{x \geq 0$: x maximizes utility for some $m > 0$ at the given $p\}$. It is easy to show for the direct utility function in this question that $x \in$ IEP implies $tx \in$ IEP for all $t > 0$. What feature of the utility function leads to this result? Give a detailed explanation.

4. Consider a pure exchange economy with consumers $i = A, B$ and goods $j = 1, 2$. Consumer A has the utility function $u_A(x_A) = \ln x_{A1} + \ln x_{A2}$. Consumer B has the utility function $u_B(x_B) = k_1 x_{B1} + k_2 x_{B2}$ where $k_1 > 0, k_2 > 0$. Assume $x_A \geq 0$ and $x_B \geq 0$. The aggregate endowments of the two goods are $w_1 = 1$ and $w_2 = 1$.

(a) Maximize A's utility subject to physical feasibility constraints and the additional constraint that B has the utility level u_B^0. Solve mathematically for the resulting allocation. Give a verbal interpretation using an Edgeworth box.

(b) A social planner maximizes $au_A(x_A) + bu_B(x_B)$ subject to physical feasibility where the weights $a > 0$ and $b > 0$ are constants. What mathematical condition must the ratio a/b satisfy if the planner's allocation is the same as the allocation in part (a)? Give a verbal interpretation using an Edgeworth box.

(c) Consider a Walrasian equilibrium for this economy. What can you say about the WE prices? What mathematical condition do the individual endowments need to satisfy in order for the WE allocation to be the same as the allocations from parts (a) and (b)? Give a verbal interpretation using an Edgeworth box.

5. Matt Damon lives alone on Mars. He cares about two goods: potatoes ($x \geq 0$) and water ($y \geq 0$). His utility function is $u = \min\{ax, by\}$ where $a > 0$ and $b > 0$. He owns a firm that can produce any output vector (x, y) with $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

(a) Draw separate graphs for $a = b$, $a > b$, and $a < b$. In each case, show the feasible outputs, a few indifference curves, and the set of optimal consumption bundles.

Now Matt wants to know whether there are prices $p \geq 0$ (for potatoes) and $q \geq 0$ (for water) such that (i) his firm is maximizing profit; (ii) he is maximizing utility subject to his budget constraint; and (iii) there is no excess demand for any good. Prices for inputs to the firm are always zero so profit will be identical to revenue. Matt has no physical endowments and his only income is the profit from his firm.

(b) Assume $a = b$. Show that any strictly positive prices $(p, q) > 0$ lead to a Walrasian equilibrium. Explain your reasoning with a graph. Can there be an excess supply of some good in WE? Why or why not?

(c) Assume $a > b$. Show that there is no WE with both prices strictly positive, and no WE with both prices equal to zero. Then show that there is a WE where one price is positive and the other is zero. Explain your reasoning using a graph. Can there be an excess supply of some good in WE? Why or why not?